## Интегриране по части

I. *Неопределени интеграли.*

**1)**
$$\int (x^3-2x^2+5)e^{3x}dx = \tfrac{1}{3}(x^3-2x^2+5)e^{3x} - \tfrac{1}{3}\int (3x^2-4x)e^{3x}dx =$$
$$= \tfrac{1}{3}(x^3-2x^2+5)e^{3x} - \tfrac{1}{9}(3x^2-4x)e^{3x} + \tfrac{1}{9}\int (6x-4)e^{3x}dx = \tfrac{1}{3}(x^3-2x^2+5)e^{3x} -$$
$$-\tfrac{1}{9}(3x^2-4x)e^{3x} + \tfrac{1}{27}(6x-4)e^{3x} - \tfrac{2}{9}\int e^{3x}dx = \tfrac{1}{9}(3x^3-9x^2+6x+13)e^{3x} + C.$$

**2)**
$$\int x^4e^{-x}dx = -x^4e^{-x} + 4\int x^3e^{-x}dx = -x^4e^{-x} - 4x^3e^{-x} + 12\int x^2e^{-x}dx =$$
$$= -(x^4+4x)e^{-x} - 12x^2e^{-x} + 24\int xe^{-x}dx = -(x^4+4x^3+12x^2)e^{-x} -$$
$$-24xe^{-x} + 24\int e^{-x}dx = -(x^4+4x^3+12x^2+24x+24)e^{-x} + C.$$

**3)**
$$\int x^5\Big/e^{x^2}\,dx = \tfrac{1}{2}\int x^4e^{-x^2}dx^2 = \tfrac{1}{2}\int_{/x^2=t/} t^2e^{-t}dt = -\tfrac{1}{2}t^2e^{-t} + \tfrac{1}{2}.2\int te^{-t}dt =$$
$$= -\tfrac{1}{2}t^2e^{-t} - te^{-t} + \int e^{-t}dt = -(\tfrac{1}{2}t^2+t+1)e^{-t} + C = -(\tfrac{1}{2}x^4+x^2+1)e^{-x^2} +$$
$$+C\ /t = x^2/.$$

**4)**
$$\int x^2.\sin x\,dx = -x^2.\cos x + 2\int x\cos x\,dx = -x^2.\cos x + 2x\sin x -$$
$$-2\int \sin x\,dx = -x^2.\cos x + 2x\sin x + 2\cos x + C = (2-x^2)\cos x +$$
$$+2x\sin x + C.$$

**5)**
$$\int (x^3-2x^2+5)\sin 3x\,dx = -\tfrac{1}{3}(x^3-2x^2+5)\cos 3x + \tfrac{1}{3}\int (3x^2-4x)\cos 3x\,dx$$
$$= -\tfrac{1}{3}(x^3-2x^2+5)\cos 3x + \tfrac{1}{9}(3x^2-4x)\sin 3x - \tfrac{1}{9}\int (6x-4)\sin 3x\,dx =$$
$$= -\tfrac{1}{3}(x^3-2x^2+5)\cos 3x + \tfrac{1}{9}(3x^2-4x)\sin 3x + \tfrac{1}{27}(6x-4)\cos 3x -$$
$$-\tfrac{2}{9}\int \cos 3x\,dx = -\tfrac{1}{3}(x^3-2x^2-\tfrac{2}{3}x+\tfrac{49}{9})\cos 3x + \tfrac{1}{9}(3x^2-4x-\tfrac{2}{3})\sin 3x + C$$
$$= -\tfrac{1}{27}(9x^3-18x^2-6x+49)\cos 3x + \tfrac{1}{27}(9x^2-12x-2)\sin 3x + C.$$

**6)**

$$\int x^3.\sin(2x+3)\,dx=\tfrac{1}{2}\int x^3.\sin(2x+3)\,d(2x+3)=-\tfrac{1}{2}\int x.^3 d\cos(2x+3)=$$
$$=-\tfrac{1}{2}x^3.\cos(2x+3)+\tfrac{3}{2}\int x^2.\cos(2x+3)\,dx=-\tfrac{1}{2}x^3.\cos(2x+3)+$$
$$+\tfrac{3}{4}x^2.\sin(2x+3)-\tfrac{3}{4}.2\int x.\sin(2x+3)\,dx=-\tfrac{1}{2}x^3.\cos(2x+3)+\tfrac{3}{4}x^2.\sin(2x+3)$$
$$+\tfrac{3}{4}x.\cos(2x+3)-\tfrac{3}{4}\int\cos(2x+3)\,dx=(\tfrac{3}{4}x-\tfrac{1}{2}x^3)\cos(2x+3)+$$
$$+(\tfrac{3}{4}x^2-\tfrac{3}{8})\sin(2x+3)+C=\tfrac{1}{4}(3x-2x^3)\cos(2x+3)+\tfrac{3}{8}(2x^2-1)\sin(2x+3)+C.$$

II. *Определени интеграли.*

**1⁰)** $$\int_0^1 xe^{-x}dx=-xe^{-x}\,|_0^1+\int_0^1 e^{-x}dx=-e^{-1}-e^{-x}\,|_0^1=1-2.e^{-1}.$$

**2⁰)**

$$\int_0^\pi x^3.\sin x\,dx=-x^3.\cos x\,|_0^\pi+3\int_0^\pi x^2.\cos x\,dx=\pi^3+3x^2.\sin x\,|_0^\pi-6\int_0^\pi x.\sin x\,dx$$
$$=\pi^3+6x.\cos x\,|_0^\pi-6\int_0^\pi\cos x\,dx=\pi^3-6\pi.$$

**3⁰)**

$$\int_0^1(x^2-2x)\cos\pi(x+1)\,dx=\tfrac{1}{\pi}\int_0^1(x^2-2x)\cos\pi(x+1)\,d\pi(x+1)=$$
$$=\tfrac{1}{\pi}(x^2-2x)\sin\pi(x+1)\,|_0^1-\tfrac{1}{\pi}2\int_0^1(x-1)\sin\pi(x+1)\,dx=\tfrac{2}{\pi^2}(x-1).$$
$$.\cos\pi(x+1)\,|_0^1-\tfrac{2}{\pi^2}\int_0^1\cos\pi(x+1)\,dx=-\tfrac{2}{\pi^2}-\tfrac{2}{\pi^3}\sin\pi(x+1)\,|_0^1=-\tfrac{2}{\pi^2}.$$

**4⁰)**

$$\int_{-1}^2(x^3+3x+5)\cos 2\pi x\,dx=\tfrac{1}{2\pi}\int_{-1}^2(x^3+3x+5)\cos 2\pi x\,d(2\pi x)=$$
$$=\tfrac{1}{2\pi}(x^3+3x+5)\sin(2\pi x)\,|_{-1}^2-\tfrac{1}{2\pi}\int_{-1}^2(3x^2+3)\sin(2\pi x)\,dx=$$
$$=\tfrac{1}{4\pi^2}(3x^2+3)\cos 2\pi x\,|_{-1}^2-\tfrac{1}{4\pi^2}\int_{-1}^2 6x.\cos 2\pi x\,dx=\tfrac{1}{4\pi^2}(15-6)-$$
$$-\tfrac{1}{8\pi^3}6x.\sin 2\pi x\,|_{-1}^2+\tfrac{6}{8\pi^3}\int_{-1}^2\sin 2\pi x\,dx=\tfrac{9}{4\pi^2}-\tfrac{6}{16\pi^4}\cos 2\pi x\,|_{-1}^2=\tfrac{9}{4\pi^2}.$$