SQEMA with Universal Modality

Dimiter Georgiev
dimitertg@yahoo.com

Sofia University "St. Kliment Ohridski"
Faculty of Mathematics and Informatics
Supported by the Science Fund of the Sofia University
and the BSF, programme Rila 2014.

10th Panhellenic Logic Symposium, June 13th, 2015

Table of Contents

Introduction

A *Kripke frame* is an ordered pair of the kind $< W, R >$, where W is a non-empty set, and $R \subseteq W \times W$ is a binary relation over W.

One one hand, Kripke frames are structures for modal formulas, but on the other hand, they are structures for a first-order language with a single predicate symbol R.

Johan van Benthem posed the question: is there a formula of this first-order language, which is valid exactly in those Kripke frames, which validate a given modal formula? And if it exists, how can it be found?

Introduction

A *Kripke frame* is an ordered pair of the kind $< W, R >$, where W is a non-empty set, and $R \subseteq W \times W$ is a binary relation over W. One one hand, Kripke frames are structures for modal formulas, but on the other hand, they are structures for a first-order language with a single predicate symbol R.

Johan van Benthem posed the question: is there a formula of this first-order language, which is valid exactly in those Kripke frames, which validate a given modal formula? And if it exists, how can it be found?

Introduction

A *Kripke frame* is an ordered pair of the kind $< W, R >$, where W is a non-empty set, and $R \subseteq W \times W$ is a binary relation over W. One one hand, Kripke frames are structures for modal formulas, but on the other hand, they are structures for a first-order language with a single predicate symbol R.

Johan van Benthem posed the question: is there a formula of this first-order language, which is valid exactly in those Kripke frames, which validate a given modal formula? And if it exists, how can it be found?

The Correspondence Problem

Given a modal formula ϕ, decide if there is a first-order formula ψ of a language with a single binary predicate symbol R, such that for every Kripke frame F: $\mathrm{F} \Vdash \phi$ iff $\mathrm{F} \vDash \psi$.

The problem was answered in Lidia Chagrova's theorem:

Theorem (L. A. Chagrova)

This problem is not algorithmically solvable.

The Correspondence Problem

Given a modal formula ϕ, decide if there is a first-order formula ψ of a language with a single binary predicate symbol R, such that for every Kripke frame F: $\mathrm{F} \Vdash \phi$ iff $\mathrm{F} \vDash \psi$.
The problem was answered in Lidia Chagrova's theorem:

Theorem (L. A. Chagrova)

This problem is not algorithmically solvable.

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas. This lead to van Benthem's question.

There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:

- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas.
This lead to van Benthem's question.
There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:
- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas.
This lead to van Benthem's question.
There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:
- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas. This lead to van Benthem's question.

There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:

- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas.
This lead to van Benthem's question.
There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:
- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas. This lead to van Benthem's question.

There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:

- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Correspondence Problem, Interesting Cases

Henrik Sahlqvist found a class of modal formulas, for which the problem has a solution. He defined the Sahlqvist class of formulas. This lead to van Benthem's question.

There are interesting cases, for which there are algorithms for finding the correspondent first-order formulas:

- The Sahlqvist-van-Benthem algorithm for Sahlqvist formulas
- SCAN (D. M. Gabbay, H. Ohlbach)
- DLS (A. Szalas)
- A method using a modal lemma by Ackermann (D. Vakarelov)
- SQEMA (D. Vakarelov, V. Goranko, W. Conradie)

The Algorithm SQEMA

Dimiter Vakarelov, later Valentin Goranko and Willem Conradie, describe an algorithm which takes a modal formula as input, not always gives a result, but when it gives a result, then the result is a predicate formula, which corresponds to the input modal formula.

If the algorithm gives a result for the modal formula ϕ, then the normal modal logic $K + \phi$ is complete, and the formula ϕ is canonical.

The algorithm always gives results for Sahlqvist formulas.

The Algorithm SQEMA

Dimiter Vakarelov, later Valentin Goranko and Willem Conradie, describe an algorithm which takes a modal formula as input, not always gives a result, but when it gives a result, then the result is a predicate formula, which corresponds to the input modal formula. If the algorithm gives a result for the modal formula ϕ, then the normal modal logic $K + \phi$ is complete, and the formula ϕ is canonical.

The algorithm always gives results for Sahlqvist formulas.

The Algorithm SQEMA

Dimiter Vakarelov, later Valentin Goranko and Willem Conradie, describe an algorithm which takes a modal formula as input, not always gives a result, but when it gives a result, then the result is a predicate formula, which corresponds to the input modal formula. If the algorithm gives a result for the modal formula ϕ, then the normal modal logic $K + \phi$ is complete, and the formula ϕ is canonical.
The algorithm always gives results for Sahlqvist formulas.

The Modal Language with the Universal Modality

We consider the basic modal language extended with the universal modality, $M(\Box, [U])$.

If $\mathcal{M} =< W, R, V >$ is a model over a Kripke frame $< W, R >$, and $w \in W$, then:

$\mathcal{M}, w \Vdash \Box\phi$ iff $\forall v \in W : wRv \Rightarrow \mathcal{M}, w \Vdash \phi$

$\mathcal{M}, w \Vdash [U]\phi$ iff $\forall v \in W : \mathcal{M}, w \Vdash \phi$

The Modal Language with the Universal Modality

We consider the basic modal language extended with the universal modality, $M(\Box, [U])$.
If $\mathcal{M} =< W, R, V >$ is a model over a Kripke frame $< W, R >$, and $w \in W$, then:
$\mathcal{M}, w \Vdash \Box\phi$ iff $\forall v \in W : wRv \Rightarrow \mathcal{M}, w \Vdash \phi$
$\mathcal{M}, w \Vdash [U]\phi$ iff $\forall v \in W : \mathcal{M}, w \Vdash \phi$

The Modal Language with the Universal Modality

We consider the basic modal language extended with the universal modality, $M(\Box, [U])$.
If $\mathcal{M} =< W, R, V >$ is a model over a Kripke frame $< W, R >$, and $w \in W$, then:
$\mathcal{M}, w \Vdash \Box\phi$ iff $\forall v \in W : wRv \Rightarrow \mathcal{M}, w \Vdash \phi$
$\mathcal{M}, w \Vdash [U]\phi$ iff $\forall v \in W : \mathcal{M}, w \Vdash \phi$

The Modal Language with the Universal Modality

We consider the basic modal language extended with the universal modality, $M(\Box, [U])$.
If $\mathcal{M} =< W, R, V >$ is a model over a Kripke frame $< W, R >$, and $w \in W$, then:
$\mathcal{M}, w \Vdash \Box\phi$ iff $\forall v \in W : wRv \Rightarrow \mathcal{M}, w \Vdash \phi$
$\mathcal{M}, w \Vdash [U]\phi$ iff $\forall v \in W : \mathcal{M}, w \Vdash \phi$

The Correspondence Problem for $M(\Box, [U])$

For the basic modal language extended with the universal modality, $M(\Box, [U])$, the correspondence problem is still algorithmically unsolvable.

We propose an extension of the SQEMA algorithm for formulas $\phi \in M(\Box, [U])$, SQEMA+U, for which it was proven that, if successful for ϕ, then:

- The result is a predicate formula, correspondent to ϕ
- The minimal normal modal logic $K_u + \phi$ is complete

The Correspondence Problem for $M(\Box, [U])$

For the basic modal language extended with the universal modality, $M(\Box, [U])$, the correspondence problem is still algorithmically unsolvable.

We propose an extension of the SQEMA algorithm for formulas $\phi \in M(\Box, [U])$, SQEMA+U, for which it was proven that, if successful for ϕ, then:

- The result is a predicate formula, correspondent to ϕ
- The minimal normal modal logic $K_u + \phi$ is complete

The Correspondence Problem for $M(\Box, [U])$

For the basic modal language extended with the universal modality, $M(\Box, [U])$, the correspondence problem is still algorithmically unsolvable.

We propose an extension of the SQEMA algorithm for formulas $\phi \in M(\Box, [U])$, SQEMA+U, for which it was proven that, if successful for ϕ, then:

- The result is a predicate formula, correspondent to ϕ
- The minimal normal modal logic $K_u + \phi$ is complete

The Correspondence Problem for $M(\Box, [U])$

For the basic modal language extended with the universal modality, $M(\Box, [U])$, the correspondence problem is still algorithmically unsolvable.

We propose an extension of the SQEMA algorithm for formulas $\phi \in M(\Box, [U])$, SQEMA+U, for which it was proven that, if successful for ϕ, then:

- The result is a predicate formula, correspondent to ϕ
- The minimal normal modal logic $K_u + \phi$ is complete

Demonstration

Demonstration

Future work

- Prove that SQEMA+U succeeds for the Sahlqvist formulas of $M(\Box, [U])$.
- Extend the algorithm and the proven results to the temporal modal language with $[U]$ and nominals.
- Implement SQEMA for modal logics with polyadic modal operators.

Future work

- Prove that SQEMA+U succeeds for the Sahlqvist formulas of $M(\Box, [U])$.
- Extend the algorithm and the proven results to the temporal modal language with $[U]$ and nominals.
- Implement SQEMA for modal logics with polyadic modal operators.

Future work

- Prove that SQEMA+U succeeds for the Sahlqvist formulas of $M(\Box, [U])$.
- Extend the algorithm and the proven results to the temporal modal language with $[U]$ and nominals.
- Implement SQEMA for modal logics with polyadic modal operators.

Thank you for your time!

Thank you for your time!